Predicting Breakdown Voltage for Microscale and Nanoscale Gaps as a Function of Pressure

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***Abstract*— The trend of electronics miniaturization for various applications, such as microelectromechanical systems (MEMS) and electric microproplusion, has necessitated more accurate predictions for breakdown voltage, V_b. Paschen's law—traditionally used for breakdown predictions—fails at gap distances smaller than ~15 μm. Modified Paschen's curves have been derived to account for this deviation, but the resulting equations must be solved numerically and do not elucidate behavior at critical limits, such as small gap distance or low pressure. We have previously obtained excellent agreement between numerical solutions, analytic solutions, and particle-in-cell (PIC) simulations at atmospheric pressure for gap distances from nanoscale to microscale. We extend this concept to sub-atmospheric pressures and demonstrate excellent agreement between numerical solutions and PIC simulations.**

I. Introduction

Recent shifts toward miniaturized devices necessitate further understanding of breakdown mechanisms within these regimes. In the microscale region, microelectromechanical systems (MEMS) are used in biotechnology, medicine, and communications for various applications including radar systems and both satellite and wireless communications [1, 2]. The microscale gaps and high operating voltages required for these devices can lead to unintentional sparking or breakdown, leading to equipment damage and failure. Applications using even smaller gaps, such as nanoelectromechanical systems (NEMS), are under investigation for enhanced sensing and scanning devices [3, 4]. Accurate breakdown voltage predictions for MEMS and NEMS can ensure safe device operation to prevent unintentional discharges [5, 6].

Conversely, one intentionally induces breakdown at microscale dimensions at various pressures for microplasmas, which are being researched for applications in medicine and environmental mitigation [7, 8]. Furthermore, microthrusters, such as colloid thrusters, laser ablation thrusters, and field emission electric propulsion systems are being developed for micropropulsion systems that aim to reduce spacecraft size [9, 10]. Accurate breakdown voltage predictions for these applications can be used for system optimization. In this pa-

per, we develop a nondimensional model to predict breakdown for nanoscale and microscale gaps as a function of pressure and compare the results to particle-in-cell simulations.

II. METHODS

Paschen's law is commonly used to predict breakdown by considering Townsend effects, or electron avalanches, as the dominant mechanism. Paschen's law determines V_b as a function of the product of gap pressure, p, and distance, d [11-12]. As the gap size decreases, too many electrons traverse the gap before causing enough ionizations for an avalanche to form, causing a sharp increase in V_b on the left-hand side of the Paschen curve minimum. However, this increase does not always occur experimentally for gaps smaller than ~15 μm [13, 14]. For smaller gaps, the electric field increases, leading to field emission, which is a tunneling effect due to sufficiently high electric fields [13]. The combination of field emission and electron avalanches reduces the breakdown voltages for smaller gaps, explaining the deviation from Paschen's curve noted experimentally by Boyle and Kisliuk in 1955 [15] and then shown mathematically and demonstrated through simulations [14, 16-25] and experiments [14, 20, 26-33].

A recently developed model couples field emission and Townsend effects [16] to derive a breakdown condition, F_{br}, as

$$F_{br} = \frac{2v_d\epsilon_o E^2\{1-\gamma_{SE}[\exp(\alpha d)-1]\}}{D_{FN}\, j_{fn}\, d\,[exp(\alpha d)-1]} = \frac{\exp(x_o)(1+2\bar{E}x_o)}{x_o}, \quad (1)$$

where γ_{SE} is the secondary electron emission coefficient, E is the electric field, $j_{FN} = C_{FN}E^2\exp(-D_{FN}/E)$, $C_{FN} = A_{FN}\beta^2\phi^{-1}t^{-2}(y)\exp\left[(3.79\times10^{-4})^2 B_{FN}/\phi^{1/2}\right]$, $D_{FN} = 0.95B_{FN}\phi^{3/2}\beta^{-1}$, β is the field enhancement factor, ϕ is the work function of the electrodes, $\alpha = C_p p\exp\left[-D_p(p/E)^{1/2}\right]$, $x_o = \left[(1+8\bar{E})^{1/2}-1\right]/4\bar{E}$, $\bar{E} = E/D_{FN}$ represents the dimensionless electric field, d is the gap distance, v_d is the ion drift velocity [16], and ϵ_o is the permittivity of free space.

To more clearly elucidate the physics occurring within the system, we followed the same process we used for atmospheric pressure [34] to nondimensionalize (1) by defining

$$\bar{E} = EE_*^{-1}, \quad \bar{T} = TT_*^{-1}, \quad \bar{p} = pp_*^{-1}, \quad \bar{d} = dL^{-1}, \quad \bar{J}_{FN} = j_{FN}j_0^{-1}, \quad \bar{\alpha} = \alpha L, \quad (2)$$

where $\bar{E}$ is the dimensionless electric field, $\bar{T}$ is the dimensionless temperature, $\bar{J}_{FN}$ is the dimensionless Fowler-Nordheim current, and $\bar{\alpha}$ is the dimensionless ionization coefficient. This yielded the following scaling parameters:

$$p_* = E_* D_p^{-2}, \quad L = p_*^{-1}C_p^{-1}, \quad j_0 = C_{FN}E_*^2, \quad E_* = D_{FN}, \quad T_* = \frac{\pi p_* m\sigma_{CE}}{E_* 2ek}\left(\frac{D_{FN}C_{FN}L}{2\epsilon_o}\right)^2. \quad (3)$$

For argon at atmospheric pressure, $p_* = 1.95\times10^4$ Torr, $L = 1.75\times10^{-8}$ m, $j_0 = 4.84\times10^{17}$ A/m^2, $E_* = 1.38\times10^9$ V/m, and $T_* = 1.06\times10^{17}$ K. The values of these scaling paraemters will vary with pressure. Substituting (2) and (3) into (1) gives

$$F_{br} = \sqrt{\frac{\bar{T}\bar{E}}{\bar{p}\bar{d}^2}} \frac{exp\left(\frac{1}{\bar{E}}\right)\left[1-\gamma_{SE}\left(\exp\left(\bar{\alpha}\bar{d}\right)-1\right)\right]}{exp\left(\bar{\alpha}\bar{d}\right)-1} = \frac{exp(x_o)(1+2\bar{E}x_o)}{x_o}, \quad (4)$$

where $\bar{T}$, $\bar{p}$, $\bar{d}$, and $\bar{\alpha}$ depend on experimental conditions. Additionally, for ease of comparison to simulations [16], $\gamma_{SE} = 0$, $\beta = 55$, and $\varphi = 5.15$ eV.

A particle-in-cell code, XPDP1, which is one-dimensional in space and three-dimensional in velocity (1D-3V) [35], was modified to include field emission effects [23, 36]. We compared the numerical results from (4) with the simulation results from XPDP1by using $V_b = \bar{V}E_*L$ to convert from dimensionless $\bar{V}$ to V_b in units of volts.

III. Results

Figure 1 compares breakdown voltage predicted from (4) to that calculated using XPDP1 for 5 nm and 5μm gaps at various pressures. At 5 nm, there is an average deviation of 3.41% between simulation results and numerical results. At 5 μm, there is an average deviation of 5.06% between numerical and simulation results.

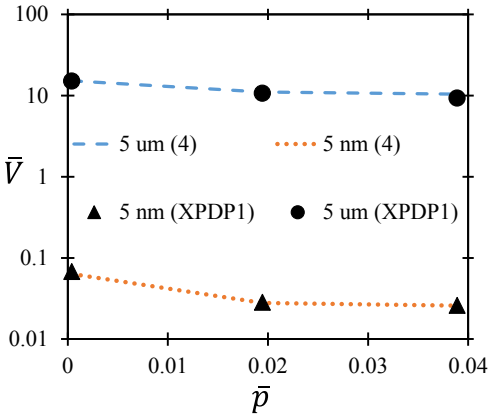

Fig. 1. Breakdown voltage determined using particle-in-cell simulations (XPDP1) and the numerical solution of (4) as a function of pressure for gap distances of 5 nm and 5 μm. The average percent deviation between (4) and XPDP1 is 3.41% at 5 nm and 5.05% at 5 μm.

The excellent agreement between simulation results and numerical results demonstrates the validity of (4) as a function of gap distance and pressure. Future work will address the effects of a nonzero γ_{SE} and will aim to simplify (4) down to analytic expressions to assess the limiting behavior of the model.

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