Analysis of a Novel Electrostatic Particle Display Device

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Abstract—**Display devices have been a growing market in the past few decades, they have been placed in everything from TV and computer screens to cell phones and refrigerator doors. No single technology has cornered the market. As a result, a lot of scientific studies have been carried out in competing technologies such as liquid crystal displays (LCD), organic light-emitting-diodes (OLED) and interferometric modulator displays (IMOD) to name a few. Another competing technology for this market is in the area of electrostatic particle displays (ESPD). For small particles, such as toner particles, the electrostatic force can be the dominate force, and this electrostatic force can be used to control the position of the particle. Gauss's law can be used to determine the attractive force on a layer of charged particles situated on a grounded electrode. By making this electrode one part of a geometrically designed capacitive cell, an electric field can then be applied to the particles to lift them off the electrode and move them outside the field of view. By placing three of these geometric cells next to each other, each containing a different primary toner color, a pixel can be created and the hue of the pixel can be varied based on the potential placed across the cells. A difficulty with this toner display technology is in the placing of these particles in and out of the field of view. This paper examines one electrode geometry and determines the lift-off voltage required to move the particles.**

I. Introduction

For the past few decades the presentation of visual information in displays by electronic means has been improving as newer display technologies develop. All the new display devices operate by applying a voltage which causes a change in a property of the device. The most common two right now are Liquid Crystal Displays (LCD) and Plasma Displays. The LCD uses an applied voltage to align liquid crystals between two pieces of conductive glass which then either allow or not allow light to pass. Plasma displays use a voltage to excite a gas to create photons of light. The interferometric modulator displays (IMOD) uses an applied voltage to flex a capacitor plate and control the reflections of light by translucent mirrors [1] and organic light-emitting-diodes (OLED) emits light when a voltage is applied across a semiconductor material. A newer device, referred to as the electrostatic particle display (ESPD) device – discussed in this paper and currently under development – uses an applied voltage to move particles in and out of the field of view to produce either reflective or transmitted light. In all these devices the quality-of-display can be characterized by graphing the applied voltage versus a property of the device that controls light output. For optimum performance good linear control of the property by the voltage is desired to allow the correct amount of light to be transmitted to the viewer.

In this paper the electric field will be determined throughout a pixel cell within the ESPD device, and then the cell will be characterized by plotting the liftoff voltage used to remove particles from the field of view versus the position of a particle in the device.

II. The ESPD Cell

A single cell of the ESPD device is shown in Fig. 1. A single cell in the device is used to make a black and white pixel. The cell consists of a planar electrode 98 μm in length

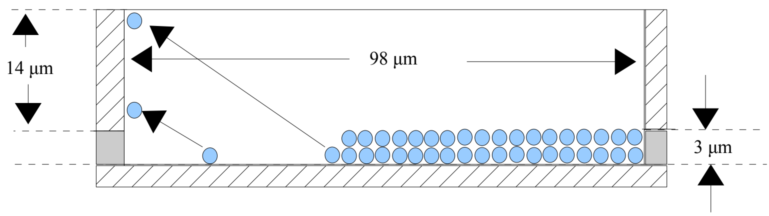

Fig. 1: Diagram showing a cell of the ESPD device under analysis

and two side electrodes – each 14 μm in height – which rest on top of 3 μm insulated spacers. In the cell there are two rows of acrylic toner particles of density 1.2 g/cm^3. At the present time the particles of interest are either 1 μm or 2.8 μm in diameter. For a color pixel there would be three such adjacent cells one for each of the three primary colors; e.g., either red, blue, and green or magenta, cyan, and yellow. By moving these particles in and out of the field of view, the complete color spectrum can be revealed.

III. Particle Physics

In charged particle physics every particle has a defined mass and charge. In order to lift charged particles off a conductive plate, the force that holds the particles to the plate needs to be determined. Once the force is determined, a counter force needs to be applied in order to lift the particles off the plate. The physics needed to determine the electrostatic adhesion of the particles to a plate is presented below. For the particle size near 1 μm the gravitational, surface tension, and van der Waals forces are assumed to be negligible compared to the electrostatic attraction force and are ignored.

A. Charged particles on a plate

The particles under study are tiny solid charged spheres similar to toner particles. When a particle is considered a sphere, the particle can then be considered a point particle. That is a particle whose mass and charge can be considered as centered at the origin of the sphere.

The permittivity of a substance is defined as

$$\varepsilon = \varepsilon_r \varepsilon_0 \tag{1}$$

where ε_r is called the dielectric constant (also referred to as the relative permittivity) which is a property of the material, and ε_0 is the permittivity of free space and equal to 8.85 x 10^{-12} F/m.

Using Gauss' Law and applying it to a uniformly charged spherical particle of diameter d_p gives the charge q_p contained in the sphere as it relates to the electric field $\boldsymbol{E}_{sp}$ on the surface of the charged particle as

$$q_p = \pi \varepsilon_0 E_{sp} d_p^2 \quad . \tag{2}$$

When a group of charged particles, each with a diameter d_p, are placed close-packed in a monolayer on a flat plane, each particle occupies an area on the plate of d_p^2. So (2) gives the charge per unit area on a flat plate due to the charged particles as

$$\frac{q_p}{d_p^2} = \pi \varepsilon_0 E_{sp} \quad . \tag{3}$$

B. The Charging of particles

When a particle is in a gas, such as air, and is passed through a region containing a charging electric field E_c and unipolar ions, these ions will be driven to the particle. As a particle receives a charge q_p, its surface field will eventually reach the value of the charging field and no further charging will take place. When a non-conductive particle has a dielectric constant ε_r some of the charging field lines will terminate on the molecule's dipoles moments. Eventually, the particles will reach the Pauthenier limit, which is the saturation charge due to field charging, and is given by [2], [3]

$$q_{p,max} = \pi \varepsilon_0 C_p E_c d_p^2 \tag{4}$$

where

$$C_p = 3\varepsilon_r / (\varepsilon_r + 2) \quad . \tag{5}$$

A typical safe charging electric field E_c is 1 MV/m, whereas a typical breakdown electric field E_{bd} is 3 MV/m. Some like to make a calculation assuming that it is possible to place particles in a charging electric field equal to the breakdown field. In other words, that $E_c = E_{bd}$. Under this condition, when these particles are placed on a flat plate, (4) and (5) give a charge per area on the flat plate of

$$\frac{q_{p,max}}{d_p^2} = \pi \varepsilon_0 C_p E_{bd} = \pi \varepsilon_0 \frac{3\varepsilon_r}{\varepsilon_r + 2} E_{bd} \quad \text{ideal; not realistic.} \tag{6}$$

Using E_{bd}=3E_c as a more practical limit of a charging field, (4) and (5) give the maximum charge per area when these particles are placed on a flat plate of

$$\frac{q_{p,max}}{d_p^2} = \frac{\pi}{3} \varepsilon_0 C_p E_{bd} = \pi \varepsilon_0 \frac{\varepsilon_r}{\varepsilon_r + 2} E_{bd} \quad \text{practical limit.} \tag{7}$$

However, from Gauss's law (3) gives the electric field at the particle's surface, and it is known that E_{sp} can not exceed the breakdown field E_{bd}. So, Gauss's law gives an upper limit of

$$\frac{q_{p,max}}{d_p^2}=\pi\,\varepsilon_0\,E_{sp}=\pi\,\varepsilon_0\,E_{bd}\quad \text{Gauss's law limit.} \tag{8}$$

Comparing (7) with (8) it can be seen that for high dielectric materials the practical charging limit reaches the Gauss's law limit, whereas for materials with a dielectric constant near unity the charge per unit area on a flat plate will be only a third of the Gauss's law limit.

A practical limit must be set such that on a charged particle, the surface field can be related to the breakdown field by

$$E_{sp}=f_{real}\,E_{bd} \tag{9}$$

where f_{real} is the fraction or ratio of the actual charge on the particle to the maximum theoretical charge possible. Hence, the maximum charge of a particle in the Pauthenier limit is given by

$$q_{p,max}=\pi\,\varepsilon_0\,f_{real}\,C_p\,E_{bd}\,d_p^2=\pi\,\varepsilon_0\,f'_{real}\,E_{bd}\,d_p^2 \tag{10}$$

where for high dielectric constant materials f'_{real} is typically found to be 0.3 [2], and

$$f'_{real}=f_{real}\,C_p\quad , \tag{11}$$

whereas, the maximum charge of a particle in the Gauss's law limit is given by

$$q_{p,max}=\pi\,\varepsilon_0\,f_{real}\,E_{bd}\,d_p^2\quad . \tag{12}$$

Since the Gauss's law limit is the limit when the corona-charging field is removed, it is the more likely physical phenomena of final charge retention and will be used in the remainder of this discussion. Hence, the charge per area on a plate due to the monolayer of charged particles is

$$\frac{q_{p,max}}{d_p^2}=\pi\,\varepsilon_0\,f_{real}\,E_{bd} \tag{13}$$

where $f_{real} \leq 1$: namely, f_{real} is unity for ideal charging and f_{real} is typically found to be 0.3 [2].

C. Adhesion of particles on a plate

The electric field due to a monolayer of particles on a flat ground plate when no external field is applied can be determined using Gauss's law. To do this, first a Gaussian surface is constructed that only covers the counter-charges in the plate. The electric field at the top of this Gaussian surface is the electric field at the bottom of the monolayer of particles. This attractive electric field is

$$E_{A,bop}=\frac{1}{\varepsilon_0}\frac{Q}{A_e}=\frac{1}{\varepsilon_0}\frac{dq}{da}=\frac{1}{\varepsilon_0}\frac{q_p}{d_p^2}=\frac{1}{\varepsilon_0}(\pi\,\varepsilon_0\,E_{sp})=\pi\,E_{sp} \tag{14}$$

Next, another Gaussian surface is constructed that covers both the monolayer of particles and the counter charges in the plate. Since the total charge is the sum of all the charges, the electric field on this Gaussian surface is zero

$$E_{A,top}=0\quad . \tag{15}$$

The average attractive electric field across the particle is

$$E_{A,e}=\frac{E_{A,top}+E_{A,bop}}{2}=\frac{\pi E_{sp}+0}{2}=\frac{\pi}{2}E_{sp}=\frac{\pi}{2}f_{real}E_{bd} \quad . \tag{16}$$

The same procedure can be used to calculate the average electric field on a second layer when two layers are present. For this situation the first Gaussian surface encloses the bottom monolayer of particles and all the counter-charges in the planar electrode. The second Gaussian surface encloses both monolayers of particles and all the counter-charges in the planar electrode. The result gives the same average attractive electric field (16) as for a single layer .

IV. Geometry of the cell

To model the cell in general terms, the cell of Fig. 1 is redrawn in Fig. 2; and the cell is now composed of a planer electrode of length L that stretches across the bottom of the cell. The height of the side electrode and spacer system is H. The height b_s is the height of the insulator or spacer. The terms a and b are the semi-major and semi-minor axis of an elliptical field line S' centered at the origin or an elliptical field line S'' centered at a distance L from the origin.

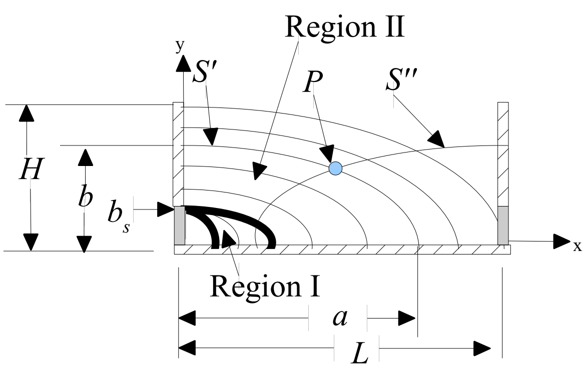

Fig. 2: A pictorial representation of an ESPD cell showing elliptical electrical field lines. All the field lines coming from the left side electrode are marked S', and the field line coming from the right electrode is marked S''. The cell has three regions; Region I is controlled by the spacer, and Region II is controlled by the *H*/*L* ratio. A third region, not analyzed, is inside an area that makes a circular arc with a radius of b_s and where very little particle movement occurs.

With the planar electrode grounded, when a voltage is applied to the side electrode, an electric field is set up within the cell. To determine the force on a charged particle located at any general point P within the cell, the electric field at the point must first be determined. If the point P is situated along the x-axis, then this electric field can be used to determine the liftoff voltage of a particle resting on the planer electrode.

When two parallel plate electrodes are charged to create a difference of potential V

across them, the electric field lines start from charges on the positive electrode and terminate on charges at the negative electrode. As Gauss's law has shown, these electric field lines enter and leave the conductive electrodes perpendicular (or normal) to the plane of the electrodes. If one of the electrodes is rotated 90 degrees, the field lines still begin and end on the charges in the electrodes. Furthermore, the field lines must still leave and enter normal to the plane of the electrodes. As a result, the field lines must follow elliptical shaped lines such as *S′* and *S″* as shown in Fig. 2.

A. Length of a field line

To describe the electric field everywhere within the ESPD cell, the equation of an ellipse centered at the origin of an *xy* plane can be used as shown in Fig. 3. The equation of an ellipse centered at the origin is

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \tag{17}$$

where a is the semi-major axis of the ellipse and b is the semi-minor axis. It will be advantageous to describe an electric field line and how it varies as a function of the distance a along the planar electrode. Referring to Fig. 3 the relationship between the radial distance r to arbitrary point $P(x, y)$ on an elliptical line is

$$r^2=x^2+y^2 \tag{18}$$

and the relation between the angle β and the distance along x is

$$x=r\sin\beta \quad . \tag{19}$$

Solving (17) for y^2 as a function of x^2 and substituting y^2 from (18) into the result, and

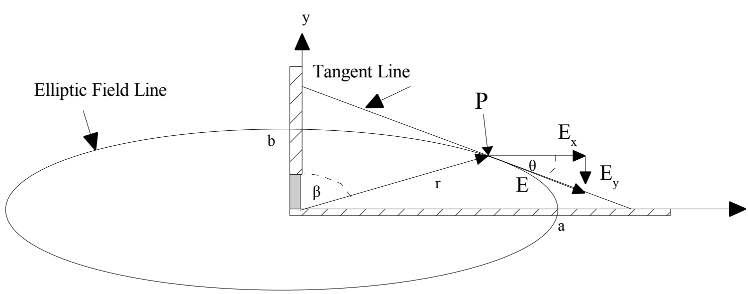

Fig. 3: An elliptical field line located in the xy has a particle at point *P* on this line a distance *r* from the origin. The angle θ is between the tangent Line and the x-axis, whereas, the angle *β* is between r and the y-axis.

then substituting x from (19) into this result and then solving the equation for r shows

$$r=\frac{a}{\sqrt{1-k^2\sin^2(\beta)}} \tag{20}$$

where

$$k=\sqrt{1-(b^2/a^2)} \tag{21}$$

and is referred to as the eccentricity of the ellipse and is always less then unity $k^2 < 1$.

Next, taking the incremental length of an ellipse $dl = r\,d\beta$ and substituting (20) for *r* and integrating over a quarter of the distance of the ellipse gives

$$\int_0^s dl = \int_0^{\frac{\pi}{2}} \frac{a}{\sqrt{1 - k^2 \sin^2(\beta)}}\, d\beta = a\, \boldsymbol{K}(k) \tag{22}$$

where S is the length of the ellipse from the side electrode at *b* to the planer electrode at *a*. This integral is called a complete elliptic integral and is known as the elliptical integral of the first kind [4]. In writing complete elliptic integrals, the eccentricity *k*, which acts as an independent variable is often omitted and ***K***(*k*) is simply written as ***K***. The series representation for ***K*** is [4]

$$\begin{aligned} \boldsymbol{K} &= \frac{\pi}{2}\left[1 + \left(\frac{1}{2}\right)^2 k^2 + \left(\frac{1\cdot 3}{2\cdot 4}\right)^2 k^4 + \ldots + \left[\frac{(2\mathrm{n}-1)!!}{2^n n!}\right]^2 k^{2\mathrm{n}} + \ldots\right] \\ &= \frac{\pi}{2}\left[1 + \sum_{n=1}^{n=\infty} \left(\frac{(2\mathrm{n}-1)!!}{2^n n!}\right)^2 k^{2\mathrm{n}}\right]. \end{aligned} \tag{23}$$

Therefore, (22) can be written as

$$S = a\,\boldsymbol{K} \tag{24}$$

with the integral representing ***K*** being replaced by (23) and where *S* is the length of the field line that terminates at a distance *a* along the *x*-axis. Another exact expression for a quarter length of the perimeter of an ellipse is given by [5]

$$S = a\frac{\pi}{2}\sum_{n=0}^{\infty} \frac{-1}{2\mathrm{n}-1}\left[\frac{(2\mathrm{n})!}{(2^n n!)^2}\right]^2 k^{2\mathrm{n}} \equiv a\,\boldsymbol{K} \tag{25}$$

where the first term ($n = 0$) is equal to 1 whereas all the others are negative correction terms. The use of (25) is more easily programmed on a spreadsheet because a spreadsheet usually contains the factorial function as one of its subroutines.

V. Define the particle on a field line.

Although it is useful to calculate the length of a line that terminates at a distance *a* along the x-axis by using (25), it is more useful to calculate the length of any line that starts on the side electrode, goes to the planar electrode and also goes through a general point *P*.

The point *P*(*x*, *y*) on a field line that crosses the *x*-axis at *a* is described by (17). The ratio

$$a/b = (H/L) \tag{26}$$

defines the shape of the elliptical field throughout Region II of the ESPD cell. If *b* in (17) is replaced by *b* in (26) and then (17) is solved for *a*, the result is

$$a = \sqrt{x^2 + (L/H)^2 (y)^2} \tag{27}$$

and if (27) is substituted into (25) the length of the electric field line that goes through point $P(x, y)$ is

$$S(x,y)=a\boldsymbol{K}=\sqrt{x^2+(L/H)^2y^2}\,\boldsymbol{K}\ . \tag{28}$$

The electric field line will be in the direction of the tangent line at the point $P(x, y)$. Taking the equation for an ellipse (17) and solving for y, then differentiating with respect to x gives

$$\frac{dy}{dx}=\frac{-bx}{a\sqrt{a^2-x^2}}=-\left(\frac{H}{L}\right)^2\frac{x}{y} \tag{29}$$

which is the slope at a point $P(x, y)$ on the perimeter of the ellipse, which is also the tangent line.

A. Location of a point

The problem can be broken up into two separate problems. In the first problem, the intersection of the left-side electrode and planar electrode are positioned at the origin of a Cartesian coordinate system as seen in Fig. 2. Then an elliptical field line S' centered at the origin is analyzed. The analysis allows a description of the x and y components of the electric field at any point on the line S' as will be discussed below.

In the second problem the intersection of the right-side electrode and planar electrode are positioned at a distance $x=L$, $y=0$ in the same Cartesian coordinate system. An elliptical field line S'' centered at $x=L$, $y=0$ is then analyzed. The analysis allows a description of the x and y components of the electrical field at any point on the line S''. If the point $P(x, y)$ lies on both lines S' and S'' as depicted in Fig. 2, then the electric fields at both lines S' and S'' can be added together using the law of vector superposition to give the total electric field at the point. Since the geometry of the cell is symmetric about a vertical line located at $L/2$, only half the cell needs to be analyzed. However, there will be an effective electric field line resulting from all the charges on the left electrode at any point P as well as an effective electric field line resulting from all the charges on the right electrode at that same point P. Using symmetry the left half and right half of the cell will be mirror reflections of each other.

1) Left Electrode

The equation of an ellipse (17) describes the path of an electric field line between two electrodes 90° apart, one situated in the xz plane the other in the yz plane. In other words, equation (17) describes any field line approaching the planar electrode at a and the left side electrode at b. The slope of (17) at the point $P(x, y)$ is given by (29). From Fig. 3 it can be seen that this slope is $\tan\theta'$, and hence

$$\theta'=\tan^{-1}[\tan(\theta')]=\tan^{-1}\left(\frac{dy}{dx}\right)=\arctan\left[-\left(\frac{H}{L}\right)^2\frac{x}{y}\right] \tag{30}$$

Finally it can be seen from Fig. 3 that

$$E'_x=E'\cos(\theta') \tag{31}$$

and

$$E'_y=E'\sin\theta' \tag{32}$$

where E' is the magnitude of the electric field at the point $P(x, y)$. The magnitude of E' is simply the voltage applied at the side electrode divided by the path length S' that transverses the distance from the left side electrode to the grounded planar electrode. This path length based on (28) is

$$S'(x, y) = a\,\boldsymbol{K} = \sqrt{x^2 + (L/H)^2 y^2}\,\boldsymbol{K} \tag{33}$$

2) Right Electrode

An elliptical curve, centered at the point (L,0) of the xy plane in a Cartesian coordinate system which has a semi-major axis a and semi-minor axis b, and goes through the point $P(x, y)$, is shown in Fig. 2 and is given by

$$\frac{(x-L)^2}{a^2} + \frac{y^2}{b^2} = 1 \quad . \tag{34}$$

Equation (34) can be solved for a which gives

$$a = \sqrt{(x-L)^2 + (L/H)^2 (y)^2} \tag{35}$$

and when (35) is substituted into (25) the final result is

$$S''(x, y) = a\,\boldsymbol{K} = \sqrt{(x-L)^2 + (L/H)^2 y^2}\,\boldsymbol{K} \quad . \tag{36}$$

Next, (34) can be solved for y, and then taking the derivative of y with respect to x gives, with the help of (26), the slope of the curve at the point $P(x, y)$ as

$$\frac{dy}{dx} = \frac{-bx}{a\sqrt{a^2 - x^2}} = -\left(\frac{H}{L}\right)^2 \frac{x-L}{y} \tag{37}$$

This slope is the tangent line at the point $P(x, y)$ and makes an angle θ'' with the x axis.

$$\theta'' = \tan^{-1}\left[\tan(\theta'')\right] = \tan^{-1}\left(\frac{dy}{dx}\right) = \arctan\left[-\left(\frac{H}{L}\right)^2 \frac{x-\lambda}{y}\right] \tag{38}$$

As a result the field components are given by

$$E''_x = E'' \cos(\theta'') \tag{39}$$

and

$$E''_y = E'' \sin(\theta'') \tag{40}$$

where E'' is the magnitude of the electric field at the point $P(x, y)$. The magnitude of E'' is simply the voltage applied at the side electrode divided by the path length S'' that transverses the distance from the right side electrode to the grounded planar electrode.

B. Three region cell

The geometry of the electrostatic cell can be broken up into three regions as shown in Fig. 2. The three regions are Region I, Region II and a third region, which is not marked and is inside an area that makes a circular arc with a radius of b_s. This third region is not analyzed because very little particle movement in the x-direction can occur in this region.

The edge of the left and right side electrodes rest on insulator spacers, and in the region where the side electrode and the insulator meet the electric field strength becomes very high. As a result the region of high electric field strength near the insulator spacer (Region I) must be treated differently than the region away from the spacer (Region II). This Region I is controlled primarily by the insulator *spacer* height. Away from Region I is Region II where the cell's general *system* structure of height H and length L determine the eccentricity of the field lines.

1) Region I

As depicted in Fig. 2, for a charged particle located in Region I there will be two field lines one from the left electrode and one from the right electrode that will sum to create the net or effective electric field at the particle. The field line coming from the right will conform to the *system* geometry ($b/a = H/L$ = constant). However, the left field line will conform to the *spacer* geometry ($b = b_s$ = constant). The eccentricity defined by (21) for the field line in Region I coming from the left electrode is no longer the constant described by (26). Instead, the eccentricity must conform to the requirement that $b = b_s$ for all field lines within Region I.

If the particle is located in Region I then the equation of an ellipse (17) must be rewritten as

$$\frac{x^2}{a^2}+\frac{y^2}{b_s^2}=1 \quad . \tag{41}$$

The effective electric field line starting at b_s and terminating at a must be determined based on both the position of the point and the height of the insulator spacer. For this situation, the position coordinates x and y and the spacer height b_s are known, so following the operations as before and solving for a gives

$$a=\frac{b_s x}{\sqrt{b_s^2-y^2}} \tag{42}$$

Thus, the eccentricity of the ellipse describing the electric field line is

$$k_s=\sqrt{1-b_s^2/a^2}=\sqrt{1-(b_s^2-y^2)/x^2} \tag{43}$$

where $x \geq b_s$. If $x \leq b_s$ then elliptical field lines have their semi-major axis along y and not x as stated earlier. This defines the third region and is not analyzed in this paper.

By combining (25) and (42) the length of the field line in Region I is

$$S_s'=a\,\boldsymbol{K}_s=\frac{b_s x}{\sqrt{b_s^2-y^2}}\boldsymbol{K}_s \quad . \tag{44}$$

The slope of the line at $P(x, y)$ is determined by solving (41) for y and then differentiating with respect to x to give

$$\frac{dy}{dx}=-\frac{b_s^2-y^2}{xy} \quad . \tag{45}$$

The slope of (41) at the point $P(x, y)$ is given by (45), and, as before, is $\tan\theta_S'$, so

$$\theta_s^{'} = \tan^{-1}\left[\tan\left(\theta_s^{'}\right)\right] = \tan^{-1}\left(\frac{dy}{dx}\right) = \arctan\left(-\frac{b_s^2 - y^2}{xy}\right) \quad . \tag{46}$$

Finally, it can be seen from Fig. 2 that

$$E_x^{'} = E^{'}\cos\left(\theta_s^{'}\right) \tag{47}$$

and

$$E_y^{'} = E^{'}\sin\theta_s^{'} \tag{48}$$

where E' is the magnitude of the electric field at the point $P(x, y)$.

Due to symmetry of the cell, only the left half side of the cell needs to be analyzed. For a charged particle located in Region I the field lines will conform to the *spacer* geometry ($b = b_s =$ constant), the left field line follows path S_s' and can be described by (44), (45), (46), (47), and (48). The second field line follows path S'' and conforms to the *system* geometry ($b/a = H/L =$ constant). Any field line coming from the right electrode can be described by (36), (37), (38), (39) and (40).

2) Region II

For a charged particle located in Region II both field lines (which follow paths S' and S'') will conform to the *system* geometry (26). Any field line coming from the left electrode can be described by (28), (29), (30), (31) and (32). Any field line coming from the right electrode can be described by (36), (37), (38), (39) and (40).

VI. Electric fields in the ESPD cell

As mentioned in the introduction, one of the goals of this paper was to determine the electric field throughout the ESPD cell. The electric field **E** at any point $P(x, y)$ is composed of two components $\mathbf{E}_x$ and $\mathbf{E}_y$ and is written as

$$\mathbf{E} = \mathbf{E_x} + \mathbf{E_y} = E_x\hat{\mathbf{x}} + E_y\hat{\mathbf{y}} \quad . \tag{49}$$

The procedure to obtain the values of E_x and E_y are discussed below.

A. Electric field in Region I

The electric field **E** at any point $P(x, y)$ in the ESPD cell is composed of two electric fields, **E**' coming from the left electrode and **E**'' coming from the right electrode. The method to obtain the explicit values of E_x and E_y in (49) is

$$\begin{aligned}
\mathbf{E} &= \mathbf{E}^{'} + \mathbf{E}^{''} \\
&= E_x^{'}\hat{\mathbf{x}} + E_y^{'}\hat{\mathbf{y}} + E_x^{''}\hat{\mathbf{x}} + E_y^{''}\hat{\mathbf{y}} \\
&= \left(E_x^{'} + E_x^{''}\right)\hat{\mathbf{x}} + \left(E_y^{'} + E_y^{''}\right)\hat{\mathbf{y}} \\
&= \left(E_x^{'}\cos\theta_s^{'} + E_x^{''}\cos\theta^{''}\right)\hat{\mathbf{x}} + \left(E_y^{'}\sin\theta_s^{'} + E_y^{''}\sin\theta^{''}\right)\hat{\mathbf{y}} \\
&= V\left(\frac{\cos\theta_s^{'}}{S_s^{'}} + \frac{\cos\theta^{''}}{S^{''}}\right)\hat{\mathbf{x}} + V\left(\frac{\sin\theta_s^{'}}{S_s^{'}} + \frac{\sin\theta^{''}}{S^{''}}\right)\hat{\mathbf{y}}
\end{aligned} \tag{50}$$

In Region I, θ_s' is defined by (46) for the left electrode and θ'' is defined by (38) for the right electrode. The value of S_s' is defined in (44) and S'' is defined by (36).

B. Electric Field in Region II

To get the electric field at any point $P(x, y)$ in Region II simply replace S_s' with S' and θ_s' with θ' in equation (50). So, in Region II the value of S' is defined by (28) for the left electrode and the value of S'' is defined by (36) for the right electrode. The value of θ' is defined by (30) for the left electrode and θ'' is defined by (38) for the right electrode.

VII. Lift-off voltage vs planar electrode distance

Applying a voltage V to the electrodes give rise to an electric field **E** defined by (50) at any point $P(x, y)$ in the cell. The adhesion force is normal to the planar electrode, so the y component of the electric field created by the applied voltage must be greater than the electrostatic adhesion force (16) to lift the particle off the plate, i.e.,

$$E_y > E_{A,e} \tag{51}$$

If the point $P(x, y) = P(x, 0)$ is on the planar electrode, then the angles θ' and θ'' will be 90 degrees and the electric fields are in the y direction. However, the field lines that go through the center of mass of a particle resting on the planer electrode will be will be at the point $P(x, y) = P(x, d_p/2)$ and E_y will be slightly dependent on θ' and θ''. Fortunately, this can be taken into account because equation (50) has the angles.

If a charged particle is resting on the planar electrode in Region I, then using (16) and using the y component of (50) into (51) and solving for V gives the liftoff voltage as

$$V > \frac{\frac{\pi}{2} f_{real} E_{bd}}{\left(\frac{1}{S_s'} + \frac{1}{S''}\right)} = \frac{a_L \boldsymbol{K_s} a_R \boldsymbol{K}}{2(a_L \boldsymbol{K_s} + a_R \boldsymbol{K})} \pi f_{real} E_{bd} \tag{52}$$

where a_L is the value from (42) and a_R is the value from (35). The values of a_L and a_R must be evaluated at the point $P(x, y) = P(x, d_p/2)$.

If a charged particle is resting on the planar electrode in Region II, then using (16) and using the y component of (50) into (51) and solving for V gives the liftoff voltage as

$$V > \frac{\frac{\pi}{2} f_{real} E_{bd}}{\left(\frac{\sin\theta'}{S'} + \frac{\sin\theta''}{S''}\right)} = \frac{a_R a_L \pi f_{real} E_{bd}}{2(a_R \sin\theta' + a_L \sin\theta'')} \boldsymbol{K} \tag{53}$$

where a_L is the value from (27) and a_R is the value from (35).

A. Results

The main purpose of this paper is to determine the liftoff voltage as a function of position along the planar electrode. To do this, the particle position along the planar electrode must be known. Also, the liftoff force must go through the center of the particle.

A uniformly charged spherical particle can be treated as a point particle with its center of mass and center of charge located at the center of the particle. While resting on the planar electrode, its center of mass and center of charge is located at $y = 0.5d_p$ above the planar electrode. If the particles are resting as a monolayer on the planar electrode, then the n^{th} particle is located at a distance $x = d_p(n\text{-}0.5)$ from the left side electrode. The n^{th} particle will be located at the point $P(x, y) = P([n-0.5]d_p, 0.5d_p)$. Equations (52) and (53) can be used to determine the liftoff voltage at any point $P(x, y)$ within the cell. For particles on the planar electrode the liftoff voltage as a function the distance from the left electrode is shown in Fig. 4.

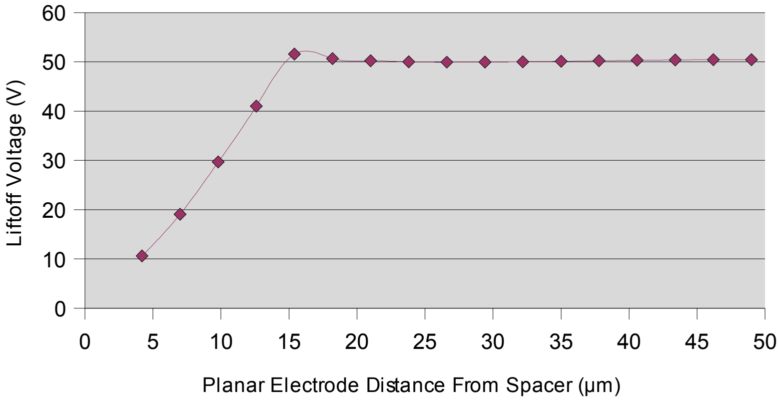

Fig. 4: Liftoff voltage as a function distance from left side electrode of the cell in Fig. 1 for 2.8 μm diameter particles charged to 30% of maximum (f_{real} = 0.3). Only data from the left half of the cell is shown as the right half is the mirror image.

Each data point in Fig. 4 represents a particle. The first particle is located in the third region and is not plotted. It can be seen in Fig. 4 that there is good linear control of particle liftoff for the second through sixth particles from the left electrode, after which there is hardly any control of the particles. The ratio given in (26) determines the field lines throughout most of the cell, which is essentially all of Region II in Fig. 2. The demarcation line in Fig. 2 which separates Region I from Region II is where $b = b_s$. Using the values for H, L and b_s given in Fig. 1 and substituting them into equation (26) gives an ellipse crossing the x-axis at approximately 17 μm. As a result, essentially all the linear control of the particles occurs in Region I. Thus, the next generation cell needs to have a larger insulating spacer to get more linear control of the liftoff of particles.

VIII. Conclusions

In this paper it was shown that a novel electrostatic particle display (ESPD) device – having the cell geometry depicted in Fig. 1 – could be analyzed using elliptical integrals of the first kind to determine the electrical field at any point in the cell. The cell could be broken up into three regions. A spacer region with a very high electric field (Region I) and a system geometry region of lower electric field (Region II) and a third region in the corners of the cell – that for the present geometry did not need to be analyzed. Region I produced good linearity between the liftoff voltage and particle position. However, Region II has poor linearity. The next generation cell needs to have a larger insulating spacer to get better linear control of liftoff of particles throughout the ESPD cell.

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